u = superficial velocity (m/s)
u_o = superficial velocity at reactor entrance (m/s)
t = time (s)
t' = transformed time (s)
ΔE = deviation from true activation energy (kcal/gmol)
Δt, $\Delta t'$ = incremental time, incremental transformed time (s)
Δx = incremental reactor distance (m)

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Mechanical Mixing Efficiency Parameter for Static Mixers

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INTRODUCTION

Static mixers or motionless mixers (Middleman, 1977, p. 327) are a special type of mixer that achieve intermaterial area (a_v) increase (Ottino et al., 1979) without any moving parts. Static mixer uses include both laminar and turbulent flow regimes (Koch bulletin KSM-2), mixing of high and low viscosity fluids, such as in-line blending of solvents, polymer blending, gas-gas mixing, and dispersion of immiscible fluids. Our description here is restricted to mixing of similar fluids in laminar flows. These types of mixers are designed to produce increase in intermaterial area per unit volume, a_v (reduction in striation thickness, s), by flow reorientation and material subdivision in such a way that a_v increases predictably in each of the repetitive flow units of the mixer. The consecutive flows are produced by a periodic pattern of non-moving solid surfaces inserted in the flow.

The repetitive flow pattern implies that flow parameters follow some kind of periodic function in which inverse time decay of stretching efficiency by shear (Ottino et al., 1981) is avoided or kept at some constant average value. It is apparent that degree of subdivision and pressure drop are inversely related. The cost of high intermaterial area generation is large pressure drops. Presently, there is no criterion for a rational comparison of these competitive effects. Here we propose to use a recently developed mixing theory for such a comparison.

THEORY

A structured continuum mixing description is based on a lamellar structure assumption (Ottino et al., 1979) and a derived point description of the state of mixing based on intermaterial area per unit volume, a_v. The transport equation for the property a_v is given as

$$\frac{\partial \ln a_v}{\partial t} + v \cdot \operatorname{grad}(\ln a_v) = e(x,t)(D{:}D)^{1/2} \tag{1}$$

where $e(x,t)$ is a mixing efficiency such that $e(x,t) < 1$. For a continuous flow mixer in steady state Eq. 1 produces the bound (Ottino et al., 1981)

$$\frac{d}{dz} \ll \ln a_v \gg \, < \frac{\overline{(D{:}D)}^{1/2}}{\bar{v}_z}, \tag{2}$$

where the mean values $\ll \gg$ and — of any function $f(x,t)$ are defined as

$$\ll f \gg \, = \frac{\int_A f v_n dA}{\int_A v_n dA}, \quad \bar{f} = \frac{\int_A f dA}{\int_A dA}, \tag{3}$$

and where A denotes the cross sectional area of the mixer. A mechanical mixing efficiency parameter, $\overline{\mathrm{eff}(z)}$, is defined with respect to the upper bound (Eq. 2) as

$$\frac{d}{dz} \ll \ln a_v \gg \, = \overline{\mathrm{eff}(z)}\, \frac{\overline{(D{:}D)}^{1/2}}{\bar{v}_z} \tag{4}$$

$\overline{\mathrm{eff}(z)}$ can be roughly interpreted as a ratio of energy used to create intermaterial area to energy dissipated by viscous action (Ottino et al., 1981).

Applications to macroscopic systems are done by relating $\overline{D{:}D}$ with the so-called viscous dissipation per unit volume per unit time, $\tau{:}D$, by means of a constitutive equation representing the rheological behavior of the structured continuum. Viscous dissipation is a bound of mechanical mixing. Values of $\overline{\tau{:}D}$ can be obtained from macroscopic balances or experimental measurements.

APPLICATION TO STATIC MIXERS

The theory will be applied to compare two commonly used static mixers: a Kenics Static Mixer and an ISG Ross Mixer (Schott et al., 1975). The Kenics Static Mixer consists of a circular pipe within which are fixed a series of short helical elements of alternating left and right-hand pitch. Due to this geometry it is possible to achieve an intermaterial area output (adapted from Tadmor and Gogos *op. cit.*, p. 442) given approximately by

$$\frac{\ll a_v \gg}{\ll a_{v_o} \gg} \simeq 2^N \tag{5}$$

where N is the number of flow units the material has passed through. Since we refer to intermaterial area delivered it is natural to use a mixed cup average $\ll \gg$, definition 3. The ISG Ross mixer using another geometry for flow reorientation and subdivision delivers intermaterial area according to (Tadmor and Gogos *op. cit.*, p. 441)

$$\frac{\ll a_v \gg}{\ll a_{v_o} \gg} \simeq 4^N \tag{6}$$

where N is the number of flow units.

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Pressure drops in static mixers are usually expressed with reference to an empty tube of the same diameter carrying the same flow rate. Experiments provide relations of the type

$$\frac{\Delta p \text{ mixer}}{\Delta p \text{ tube}} = \tilde{K}_n(Re) \tag{7}$$

where $\tilde{K}_n$ is a constant for a given Reynolds number. Constants $\tilde{K}_n$ are in the order 3-5 for Kenics mixers and 160 for ISG mixers for Reynolds numbers of practical interest (Middleman, 1977, p. 331 and ff.) and increase slightly with increasing Reynolds numbers.

Static mixers are a subset of continuous mixers and as such, generation of intermaterial area is governed by Eq. 4. Using the Jensen inequality (Hardy et al., 1973) we obtain $ln \ll a_v \gg\ >\ \ll ln\, a_v \gg$. Assuming $ln \ll a_v \gg\ \simeq\ \ll ln\, a_v \gg$ Eq. 4 is written as

$$\frac{d}{dz} ln \ll a_v \gg\ \simeq \overline{\text{eff}(z)}\,\frac{\overline{(D{:}D)^{1/2}}}{\bar{v}_z}. \tag{8}$$

Alternatively Eq. 8 can be considered to *define* $\overline{\text{eff}(z)}$ (Ottino and Macosko, 1980).

For power law fluids we have

$$\tau = 2\mu(2D{:}D)^{(n-1)/2}D, \tag{9}$$

and

$$\tau{:}D = 2^{(n+1)/2}\,\mu(D{:}D)^{(n+1)/2} \tag{10}$$

Pressure drops in static mixers (represented by subscript m) and empty tubes (represented by subscript t) are related by (cf. Eq. 7)

$$\tilde{K}_n\frac{\Delta p_t}{\rho} = \frac{K_n}{w_t}\int_0^z 2^{n+1/2}\mu\overline{(D{:}D)_t^{(n+1)/2}}A\,dz = $$
$$= \frac{\Delta p_m}{\rho} = \frac{1}{w_m}\int_0^z 2^{(n+1)/2}\mu\overline{(D{:}D)_m^{(n+1)/2}}A(z)\,dz \tag{11}$$

where the overbars represent cross sectional area average and $A(z)$ is the free cross sectional area in the static mixer and $z = Nl$. This area is nearly a constant for many static mixers (e.g. Kenics Mixer) and $A(z) \simeq A$, if not some appropriate average should be used. Since $\overline{(D{:}D)_t^{(n+1)/2}}$ is constant and Δp_t is a linear function of z, it is then clear that the term $\overline{(D{:}D)_m^{(n+1)/2}}$ is either a constant or a periodic function of z. This is a consequence of the periodic nature of the flow.

By Eq. 11 over any number of units N, where $z = Nl$,

$$\tilde{K}_n\overline{(D{:}D)_t^{(n+1)/2}} = \overline{(D{:}D)_m^{(n+1)/2}}, \tag{12}$$

and approximately

$$\tilde{K}_n^{2/(n+1)}(\overline{D{:}D})_t \simeq (\overline{D{:}D})_m. \tag{13}$$

We note that

$$\frac{\overline{(D{:}D)_t^{(n+1)/2}}}{(\overline{D{:}D})_t^{(n+1)/2}} = F(n) = \frac{2^{(1-n)/2}}{n^n}\frac{(2n(n+1))^{(n+1)/2}}{(3n+1)} \tag{14}$$

is nearly unity for $0.5 \le n \le 1.5$. Some values of $F(n)$ are listed below.

n	0.2	0.5	0.7	1.0	1.2	1.5
$F(n)$	0.733	0.911	0.959	1.000	1.016	1.032

A similar behavior is expected from $(\overline{D{:}D})_m$.

The term $(\overline{D{:}D})_t$ can be calculated for a Poiseuille flow of a power law fluid to give:

$$(\overline{D{:}D})_t = \frac{(3n+1)^2}{2n(n+1)}\frac{\langle v\rangle^2}{R^2} \tag{15}$$

where $\langle v\rangle$ is the mean velocity and R the radius of the tube. Clearly $\langle v\rangle = \bar{v}_z$ so

$$\frac{(\overline{D{:}D})_m^{1/2}}{\bar{v}_z} \simeq \frac{\tilde{K}_n^{1/(n+1)}}{R}\frac{(3n+1)}{(2n(n+1))^{1/2}} \tag{16}$$

Combining Eqs. 8 and 16 we have

$$ln\frac{\ll a_v(z)\gg}{\ll a_v(0)\gg} \simeq \langle\overline{\text{eff}(z')}\rangle_0^z\tilde{K}_n^{1/(n+1)}\frac{(3n+1)}{(2n(n+1))^{1/2}}\frac{z}{R} \tag{17}$$

where $\langle\overline{\text{eff}(z')}\rangle_0^z$ represents the mean efficiency parameter for a mixer of length z. Equation 17 is the key equation that we will use to compare static mixer performance according to Eq. 8. Since static mixers are built in such a way that delivered intermaterial area grows geometrically with the number of units (Eqs. 5 and 6), we conclude that the mean efficiency parameter is not a function of z. Combination of Eqs. 5, 6, and 17 produces

$$\langle\overline{\text{eff}(z')}\rangle_0^z \simeq \frac{(2n(n+1))^{1/2}}{(3n+1)}\frac{R}{l}\frac{ln\, M}{\tilde{K}_n^{1/(1+n)}} \tag{18}$$

where $M = 2$, $R/l = 0.31$, $\tilde{K}_n = 5(0.61)^{1-n}$ for Kinetics Mixers and $M = 4$, $R/l = 0.46$, $\tilde{K}_n = (1/4)^n(5)^{1+3n}$ for ISG Ross Mixers (Middleman, pp. 336–8). A single number quantifies the operation of the mixer.

Some values of mean efficiency are:

	$n = 0.2$	$n = 0.5$	$n = 1.0$	$n = 1.2$
Kenics Mixer	0.034	0.042	0.048	0.049
ISG Ross Mixers	0.040	0.034	0.025	0.023

Thus the Kenics Mixer is more effective than the ISG Mixer for Newtonian and dilatant fluids whereas the ISG is more effective than the Kenics Mixer for highly pseudoplastic fluids (in the sense of Eq. 4). One should note that low energy efficiencies are characteristic of all mixers (see Ottino and Macosko, 1980). Mixing of fluids is a costly operation. However, the comparison of efficiency indices has a much more practical meaning. From Eq. 8 $\overline{\text{eff}(z)}$ is interpreted as

$$\overline{\text{eff}(z)} = \frac{\dfrac{\bar{v}_z}{\ll a_v\gg}\dfrac{d \ll a_v\gg}{dz}}{(\overline{D{:}D})_m^{1/2}} \tag{19}$$

From Eq. 11 and the approximation 14 we conclude that

$$(\overline{D{:}D})_m^{1/2} = \frac{1}{\sqrt{2}}\left(\frac{\text{energy/time}}{\mu A z}\right)^{1/(n+1)} \tag{20}$$

and by combination with Eq. 19

$$\overline{\text{eff}(z)} = \frac{\dfrac{\bar{v}_z}{\ll a_v\gg}\dfrac{d \ll a_v\gg}{dz}}{\dfrac{1}{\sqrt{2}}\left(\dfrac{\text{energy/time}}{\mu A z}\right)^{1/(n+1)}} \tag{21}$$

Thus for example, for any two static mixers A and B mixing the same Newtonian fluid and such that $A_A = A_B$, $\bar{v}_{z_A} = \bar{v}_{z_B}$, $z_A = z_B$ we find that

$$\frac{\overline{\text{eff}(z)}_A}{\overline{\text{eff}(z)}_B} = \frac{\text{mixing generation}_A}{\text{mixing generation}_B}\frac{(\text{energy/time})_A^{1/2}}{(\text{energy/time})_B^{1/2}}$$

REMARK

The calculated mean efficiency index is based on average values of the stretching tensor calculated as $(\overline{D{:}D})^{1/2}$. Efficiency indices can also be defined with respect to $\overline{D{:}D)^{1/2}}$ (Ottino et al., 1981) resulting in larger numerical values as a consequence of the Hölder inequality (Hardy et al., 1973). It is instructive to compare both terms for a simple case, e.g. steady Poiseuille flow of a power law fluid in a pipe of radius R. We calculate

$$\frac{\overline{(D{:}D)^{1/2}}}{(\overline{D{:}D})^{1/2}} = G(n) = \frac{(4n(n+1))^{1/2}}{(2n+1)}$$

n	0.5	0.7	1.0	1.2	1.5
$G(n)$	0.866	0.909	0.943	0.956	0.967

Thus for Newtonian fluids the efficiencies will differ by 6%. The use of $(\overline{D{:}D})^{1/2}$ as a basis is more convenient for computation (Ottino and Macosko, 1980). However, either can be used, the only restriction being to choose a single basis for mixer comparison.

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NOTATION

$A(z)$ = cross-sectional area in continuous mixer (m^2)
a_v = intermaterial area per unit volume (m^{-1})
D = stretching tensor, (gradv) + $(\text{gradv})^T$, (s^{-1})
$e(x,t)$ = mixing efficiency, dimensionless
$eff(z)$ = mixing efficiency, dimensionless
l = length of mixing element in static mixer (m)
n = power law parameter, dimensionless
p = pressure (Pa)
t = time (s)
v = velocity vector (m/s)
V = volume of the mixer (m^3)
$\bar{v}_z$ = mean velocity at distance z($m \cdot s^{-1}$)
w = mass flow rate ($kg \cdot s^{-1}$)
z = axial distance (m)

Greek Letters

ρ = density ($kg \cdot m^{-3}$)
τ = viscous part of the stress tensor (Pa)

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Drop Breakup in the Flow of Immiscible Liquids Through an Orifice in a Pipe

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In the course of experiments on the size distribution of drops in turbulent pipe flow, we encountered what we thought were some anomalously small drops. It was conjectured that the drops might have been caused by small protrusions of gaskets at the flanged joints, and this led to an investigation of drop breakup at orifices. The investigation was a very limited one, but the results are dramatic, and we think it worthwhile to report them.

The investigation was limited to a study of the conditions under which drops of one immiscible liquid in another would break up upon flowing through a concentric orifice, i.e., a study of the percent of drops that break up in one pass through an orifice of liquid-liquid systems at a low holdup (volume fraction of drops). We did not undertake a study of drop sizes that result from the breakup.

The most closely related study we have found is that of Scott, Hayes, and Holland (1958), who measured the surface area of dispersions of water in kerosene. They correlated the results empirically and found that the surface area was proportional to the 0.735 power of the change in kinetic energy (or pressure drop) across the orifice. Thus for a given volume of dispersed phase, the Sauter mean drop diameter was inversely proportional to the 0.37 power of the orifice pressure drop.

APPARATUS AND PROCEDURE

The apparatus consisted of a 3.81 cm diameter, straight, glass pipe 25 m long. Deaerated water from a constant head tank formed the continuous phase, and the dispersed phase was a mixture of carbon tetrachloride (stabilized with 0.5% cyclohexene) and 2,2,4-trimethyl pentane saturated with water, $\rho = 0.9962$ g/cm^3, $\mu = 0.0061$ poise, $\sigma = 43$ mN/m. The temperature was kept at 23 ± 1°C. Drops of a uniform, accurately known, controllable size were made by pumping the dispersed phase with a calibrated syringe pump through a nozzle that could be impulsively flexed at regular intervals. About 20 drops of a specified size were made near the inlet at a low water flow rate so that drops were spaced at 15–50 cm intervals. The water flow was then rapidly increased to a final specified velocity and the mixture passed through an orifice located 18 m from the drop formation point. Orifices were square-edged and made of Teflon 1.6 mm thick. After all or a portion of the drops passed through the orifice, the flow was stopped. The drop fragments drifted upward against the pipe wall and were measured in the longitudinal direction with a six power magnifying device to an accuracy of about ±0.1 mm. The number of drops originally present and the percent breakup were calculated from the total volume of the fragments and the initial drop diameter. Further details are given by Percy (1969).
